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Constraining New Models with Precision Electroweak Data

Mu-Chun Chen

*Theoretical Physics Department, Fermi National Accelerator Laboratory
 Batavia, IL 60510, USA
 mcchen@fnal.gov*

Sally Dawson and Tadas Krupovnickas

*High Energy Theory Group, Brookhaven National Laboratory
 Upton, New York 11973, U.S.A.
 dawson@quark.phy.bnl.gov, tadas@quark.phy.bnl.gov*

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Electroweak precision data have been extensively used to constrain models containing physics beyond that of the Standard Model. When the model contains Higgs scalars in representations other than singlets or doublets, and hence $\rho \neq 1$ at tree level, a correct renormalization scheme requires more inputs than the three commonly used for the Standard Model case. In such cases, the one loop electroweak results cannot be split into a Standard Model contribution plus a piece which vanishes as the scale of new physics becomes much larger than M_W . We illustrate our results by presenting the dependence of M_W on the top quark mass in a model with a Higgs triplet and in the $SU(2)_L \times SU(2)_R$ left-right symmetric model. In these models, the allowed range for the lightest neutral Higgs mass can be as large as a few TeV.

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1. Introduction

Measurements at LEP, SLD, and the Tevatron have been used extensively to limit models with physics beyond that of the Standard Model (SM)¹. By performing global fits to a series of precision measurements, information about the parameters of new models can be inferred^{2,3}. The simplest example of this approach is the prediction of the W boson mass. In the Standard Model, the W - boson mass, M_W , can be predicted in terms of other parameters of the theory. The predicted W boson mass is strongly correlated with the experimentally measured value of the top quark mass, m_t , and increases quadratically as the top quark mass is increased. This strong correlation between M_W and m_t in the Standard Model can be used to limit the allowed region for the Higgs boson mass⁴.

In a model with Higgs particles in representations other than $SU(2)$ doublets and

singlets, there are more parameters in the gauge/Higgs sector than in the Standard Model. The SM tree level relation, $\rho = M_W^2/(M_Z^2 c_\theta^2) = 1$ no longer holds and when the theory is renormalized at one loop, models of this type will require extra input parameters^{5,6,7}. Models with new physics are often written in terms of the SM Lagrangian, L_{SM} plus an extra contribution,

$$L = L_{SM} + L_{NP} \quad (1)$$

where L_{NP} represents contributions from new physics beyond the SM. Phenomenological studies have then considered the contributions of L_{SM} at one-loop, plus the tree level contributions of L_{NP} . In this note, we give two specific examples with $\rho \neq 1$ at tree level, where we demonstrate that this procedure is incorrect. We discuss in detail what happens in these models when the scale of the new physics becomes much larger than the electroweak scale and demonstrate explicitly that the SM is not recovered.

The possibility of a heavy Higgs boson which is consistent with precision electroweak data has been considered by Chivukula, Hoelbling and Evans⁸ and by Peskin and Wells⁹ in the context of oblique corrections. In terms of the S , T and U parameters^{2,3}, a large contribution to isospin violation, $\delta\rho = \alpha T > 1$, can offset the contribution of a heavy Higgs boson to electroweak observables such as the W boson mass. The triplet model considered in this paper provides an explicit realization of this mechanism. The oblique parameter formulation neglects contributions to observables from vertex and box diagrams, which are numerically important in the example discussed here.

In Section 2, we review the important features of the SM for our analysis. We discuss two examples in Sections 3 and Appendix Appendix C where the new physics does not decouple from the SM at one-loop. For simplicity, we consider only the dependence of the W boson mass on the top quark mass and demonstrate that a correct renormalization scheme gives very different results from the SM result in these models. Section 3 contains a discussion of the SM augmented by a real scalar triplet, and Appendix Appendix C contains a discussion of a left-right $SU(2)_L \times SU(2)_R$ symmetric model. In Section 4, we show that the dependence on scalar masses in the W -boson mass is quadratic and demonstrate that the triplet is non-decoupling. Our major results are summarized in Eq. 31-33. These results are novel and have not been discussed in the literature before. Section 5 contains our numerical results and Section 6 concludes this paper. Similar results in the context of the littlest Higgs model have previously been found in Ref. 10, 11.

2. Renormalization

The one-loop renormalization of the SM has been extensively studied^{12,13,14} and we present only a brief summary here, in order to set the stage for Sections 3 and Appendix Appendix C. In the electroweak sector of the SM, the gauge sector has three fundamental parameters, the $SU(2)_L \times U(1)_Y$ gauge coupling constants,

g and g' , as well as the vacuum expectation (VEV) of the Higgs boson, v . Once these three parameters are fixed, all other physical quantities in the gauge sector can be derived in terms of these three parameters and their counter terms^a. We can equivalently choose the muon decay constant, G_μ , the Z-boson mass, M_Z , and the fine structure constant evaluated at zero momentum, $\alpha \equiv \alpha(0)$, as our input parameters. Experimentally, the measured values for these input parameters are⁴,

$$G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2} \quad (2)$$

$$M_Z = 91.1876(21) \text{ GeV} \quad (3)$$

$$\alpha = 1/137.036 \quad . \quad (4)$$

The W-boson mass then can be defined through muon decay^{12,15},

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 s_\theta^2} \left[1 + \Delta r_{SM} \right] \quad (5)$$

where Δr_{SM} summarizes the radiative corrections,

$$\Delta r_{SM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2} - \frac{\delta M_W^2}{M_W^2} \quad , \quad (6)$$

where $s_\theta = \sin \theta_W$, $c_\theta = \cos \theta_W$ and θ_W is the weak mixing angle. The SM satisfies $\rho = 1$ at tree level,

$$\rho = 1 = \frac{M_W^2}{M_Z^2 c_\theta^2} \quad . \quad (7)$$

In Eq. (7), M_W and M_Z are the physical gauge boson masses, and so our definition of the weak mixing angle, \bar{s}_θ , corresponds to the on-shell scheme¹⁶. It is important to note that in the SM, \bar{s}_θ is not a free parameter, but is derived from

$$\bar{s}_\theta^2 = 1 - \frac{M_W^2}{M_Z^2} \quad . \quad (8)$$

The counterterms of Eq. (6) are given by^{15,17},

$$\frac{\delta G_\mu}{G_\mu} = -\frac{\Pi_{WW}(0)}{M_W^2} + \delta_{V-B} \quad (9)$$

$$\frac{\delta\alpha}{\alpha} = \Pi'_{\gamma\gamma}(0) + 2\frac{\bar{s}_\theta}{c_\theta} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \quad (10)$$

$$\frac{\delta M_W^2}{M_W^2} = \frac{\Pi_{WW}(M_W^2)}{M_W^2} \quad , \quad (11)$$

where Π_{XY} , for $(XY = WW, ZZ, \gamma\gamma, \gamma Z)$, are the gauge boson 2-point functions; $\Pi'_{\gamma\gamma}(0)$ is defined as $\left. \frac{d\Pi_{\gamma\gamma}(p^2)}{dp^2} \right|_{p^2=0}$. The term δ_{V-B} contains the box and vertex contributions to the renormalization of G_μ ^{15,17}.

^aThere are of course also the fermion masses and the Higgs boson mass. The renormalization of these quantities does not affect our discussion. We assume that the contributions from Higgs tadpole graphs can be set to zero with an appropriate renormalization condition.

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The counterterm for \overline{s}_θ^2 can be derived from Eq. (7),

$$\frac{\delta \overline{s}_\theta^2}{\overline{s}_\theta^2} = \frac{\overline{c}_\theta^2}{\overline{s}_\theta^2} \left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right] = \frac{\overline{c}_\theta^2}{\overline{s}_\theta^2} \left[\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2} \right] . \quad (12)$$

Putting these contributions together we obtain,

$$\begin{aligned} \Delta r_{SM} = & \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_{\gamma\gamma}(0) + 2 \frac{\overline{s}_\theta}{\overline{c}_\theta} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \\ & - \frac{\overline{c}_\theta^2}{\overline{s}_\theta^2} \left[\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2} \right] . \end{aligned} \quad (13)$$

These gauge boson self-energies can be found in Ref. 10 and 18, 19 and we note that the fermion and scalar contributions to the two-point function $\Pi_{\gamma Z}(0)$ vanish. The dominant contributions to Δr_{SM} is from the top quark, and the contributions of the top and bottom quarks to the gauge boson self-energies are given in Appendix A. The m_t^2 dependence in $\Pi_{WW}(0)$ and $\Pi_{WW}(M_W^2)$ exactly cancel, thus the difference, $\Pi_{WW}(0) - \Pi_{WW}(M_W^2)$, depends on m_t only logarithmically. The second term, $\Pi'_{\gamma\gamma}(0)$, also depends on m_t logarithmically. However, the quadratic m_t^2 dependence in $\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}$ and $\frac{\Pi_{WW}(M_W^2)}{M_W^2}$ do not cancel. Thus Δr_{SM} depends on m_t quadratically, and is given by the well known result, keeping only the two-point functions that contain a quadratic dependence on m_t ^{20,21},

$$\begin{aligned} \Delta r_{SM}^t & \simeq -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_Z^2}{M_Z^2} - \left(\frac{\overline{c}_\theta^2 - \overline{s}_\theta^2}{\overline{c}_\theta^2} \right) \frac{\delta \overline{s}_\theta^2}{\overline{s}_\theta^2} \\ & \simeq -\frac{G_\mu}{\sqrt{2}} \frac{N_c}{8\pi^2} \left(\frac{\overline{c}_\theta^2}{\overline{s}_\theta^2} \right) m_t^2 , \end{aligned} \quad (14)$$

where $N_c = 3$ is the number of colors and the superscript t denotes that we have included only the top quark contributions, in which the dominant contribution is quadratic. The complete contribution to Δr_{SM} can be approximated,

$$\Delta r_{SM} \simeq .067 + \Delta r_{SM}^t + \frac{\alpha}{\pi \overline{s}_\theta^2} \frac{11}{48} \left(\ln \left(\frac{M_H^2}{M_Z^2} \right) - \frac{5}{6} \right) + \text{2-loop contributions} . \quad (15)$$

The first term in Eq. (15) results from the scaling of $\delta\alpha$ from zero momentum to M_Z ²². In the numerical results, the complete contributions to Δr_{SM} from top and bottom quarks, the Higgs boson as well as the gauge bosons are included, as given in Eq. (13).

3. Standard Model with an additional $SU(2)_L$ triplet Higgs boson

In this section, we consider the SM with an additional Higgs boson transforming as a real triplet ($Y=0$) under the $SU(2)_L$ gauge symmetry. Hereafter we will call this the Triplet Model (TM)^{23,24}. This model has been considered at one-loop by Blank and Hollik²⁵ and we have checked that our numerical codes are correct by reproducing their results. In addition, we derive the scalar mass dependence in this

model and show that the triplet is non-decoupling by investigating various scalar mass limits. We also find the conditions under which the lightest neutral Higgs can be as heavy as a TeV, which has new important implications on Higgs searches. These results concerning the scalar fields are presented in the next section.

The $SU(2)_L$ Higgs doublet in terms of its component fields is given by,

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi^0 + i\phi_I^0) \end{pmatrix}, \quad (16)$$

with ϕ^0 being the Goldstone boson corresponding to the longitudinal component of the Z gauge boson. A real $SU(2)_L$ triplet, Φ , can be written as (η^+, η^0, η^-) ,

$$\Phi = \begin{pmatrix} \eta^+ \\ v' + \eta^0 \\ -\eta^- \end{pmatrix}. \quad (17)$$

There are thus four physical Higgs fields in the spectrum: There are two neutral Higgs bosons, H^0 and K^0 ,

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} \phi^0 \\ \eta^0 \end{pmatrix}, \quad (18)$$

and the mixing between the two neutral Higgses is described by the angle γ . The charged Higgses H^\pm are linear combinations of the charged components in the doublet and the triplet, with a mixing angle δ ,

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}, \quad (19)$$

where G^\pm are the Goldstone bosons corresponding to the longitudinal components of W^\pm . The masses of these four physical scalar fields, M_{H^0} , M_{K^0} and M_{H^\pm} , respectively, are free parameters in the model. The W boson mass is given by,

$$M_W^2 = \frac{g^2}{4}(v^2 + v'^2), \quad (20)$$

where $v/\sqrt{2} = \langle \phi^0 \rangle$ is the VEV of the neutral component of the $SU(2)_L$ Higgs boson and $v' = \langle \eta^0 \rangle = \frac{1}{2}v \tan \delta$ is the vacuum expectation value of the additional scalar, leading to the relationship $v_{SM}^2 = (246 \text{ GeV})^2 = v^2 + v'^2$. A real triplet does not contribute to M_Z , leading to

$$\rho = 1 + 4 \frac{v'^2}{v^2} = \frac{1}{c_\delta^2}. \quad (21)$$

The main result of this section is to show that the renormalization of a theory with $\rho \neq 1$ at tree level is fundamentally different from that of the SM.

Due to the presence of the $SU(2)_L$ triplet Higgs, the gauge sector now has four fundamental parameters, the additional parameter being the VEV of the $SU(2)_L$ triplet Higgs, v' . A consistent renormalization scheme thus requires a fourth input parameter²⁵. We choose the fourth input parameter to be the effective leptonic

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mixing angle, \hat{s}_θ , which is defined as the ratio of the vector to axial vector parts of the $Z\bar{e}e$ coupling,

$$L = -i\bar{e}(v_e + \gamma_5 a_e)\gamma_\mu e Z^\mu, \quad (22)$$

with $v_e = \frac{1}{2} - 2\hat{s}_\theta^2$ and $a_e = \frac{1}{2}$. This leads to the definition of \hat{s}_θ ,

$$1 - 4\hat{s}_\theta^2 = \frac{\text{Re}(v_e)}{\text{Re}(a_e)}. \quad (23)$$

The measured value from LEP is given by $\hat{s}_\theta^2 = 0.23150 \pm 0.00016$ ¹.

As usual the W boson mass is defined through muon decay^{12,15},

$$G_\mu = \frac{\pi\alpha(M_Z)}{\sqrt{2}M_Z^2\hat{c}_\theta^2\hat{s}_\theta^2\rho(1 - \Delta r_{\text{triplet}})}, \quad (24)$$

where we have chosen $\alpha(M_Z)$ instead of $\alpha(0)$ as an input parameter. The contribution to $\Delta r_{\text{triplet}}$ is similar to that of the SM,

$$\Delta r_{\text{triplet}} = -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta\alpha}{\alpha(M_Z)} - \left(\frac{\hat{c}_\theta^2 - \hat{s}_\theta^2}{\hat{c}_\theta^2}\right)\frac{\delta\hat{s}_\theta^2}{\hat{s}_\theta^2} - \frac{\delta\rho}{\rho}, \quad (25)$$

where the counter term $\delta\rho$ is defined through M_W , M_Z and \hat{s}_θ as,

$$\frac{\delta\rho}{\rho} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} + \left(\frac{\hat{s}_\theta^2}{\hat{c}_\theta^2}\right)\frac{\delta\hat{s}_\theta^2}{\hat{s}_\theta^2}. \quad (26)$$

Unlike in the SM case where \bar{s}_θ is defined through M_W and M_Z as given in Eq. (12), now \hat{s}_θ is an independent parameter, and its counter term is given by²⁵,

$$\begin{aligned} \frac{\delta\hat{s}_\theta^2}{\hat{s}_\theta^2} &= \text{Re} \left[\left(\frac{\hat{c}_\theta}{\hat{s}_\theta} \right) \left[\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} + \frac{v_e^2 - a_e^2}{a_e} \Sigma_A^e(m_e) \right. \right. \\ &\quad \left. \left. - \frac{v_e}{2\hat{s}_\theta\hat{c}_\theta} \left(\frac{\Lambda_V^{Z\bar{e}e}(M_Z^2)}{v_e} - \frac{\Lambda_A^{Z\bar{e}e}(M_Z^2)}{a_e} \right) \right] \right], \\ &\equiv \left(\frac{\hat{c}_\theta}{\hat{s}_\theta} \right) \frac{\text{Re}(\Pi^{\gamma Z}(M_Z^2))}{M_Z^2} + \delta'_{V-B}, \end{aligned} \quad (27)$$

where Σ_A^e is the axial part of the electron self-energy, $\Lambda_V^{Z\bar{e}e}$ and $\Lambda_A^{Z\bar{e}e}$ are the vector and axial-vector form factors of the vertex corrections to the $Z\bar{e}e$ coupling. These effects have been included in our numerical results^{25,26}. The total correction to $\Delta r_{\text{triplet}}$ in this case is then given by,

$$\begin{aligned} \Delta r_{\text{triplet}} &= \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_{\gamma\gamma}(0) + 2\frac{\hat{s}_\theta}{\hat{c}_\theta} \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \\ &\quad - \frac{\hat{c}_\theta}{\hat{s}_\theta} \frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} + \delta_{V-B} + \delta'_{V-B}, \end{aligned} \quad (28)$$

where δ_{V-B} summarizes the vertex and box corrections in the TM model, and it is given by²⁵,

$$\delta_{V-B} = \frac{\alpha}{4\pi\hat{s}_\theta^2} \left[6 + \frac{10 - 10\hat{s}_\theta^2 - 3(R/\hat{c}_\theta^2)(1 - 2\hat{s}_\theta^2)}{2(1 - R)} \ln R \right], \quad R \equiv M_W^2/M_Z^2, \quad (29)$$

where we show only the finite contributions in the above equation. Keeping only the top quark contribution,

$$\frac{\delta \hat{s}_\theta^2}{\hat{s}_\theta^2} = \left(\frac{\hat{c}_\theta}{\hat{s}_\theta} \right) \frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} = -\frac{\alpha}{\pi \hat{s}_\theta^2} \left(\frac{1}{2} - \frac{4}{3} \hat{s}_\theta^2 \right) \left\{ \frac{1}{3} \left(\ln \frac{Q^2}{m_t^2} + \frac{1}{\epsilon} \right) - 2I_3 \left(\frac{M_Z^2}{m_t^2} \right) \right\} \quad (30)$$

where Q is the momentum cutoff in dimensional regularization and the definition of the function I_3 can be found in Appendix A. As $\Pi_{\gamma Z}(M_Z^2)$ is logarithmic, the m_t dependence of M_W is now logarithmic. We note that this much softer relation between M_W and m_t is independent of the choice of the fourth input parameter. This will become clear in our second example, the left-right symmetric model, given in Appendix C. In our numerical results, we have included in $\Delta r_{triplet}$ the complete contributions, which are summarized in Appendix B, from the top and bottom quarks and the four scalar fields, as well as the gauge bosons, and the complete set of vertex and box corrections.

4. Non-decoupling of the Triplet

As shown in Appendix B, $\Delta r_{triplet}$ depends on scalar masses quadratically. This has important implications for models with triplets, such as the littlest Higgs model^{10,11}. The two point function $\Pi_{\gamma Z}(0)$ does not have any scalar dependence, while $\Pi'_{\gamma\gamma}(0)$ and $\Pi_{\gamma Z}(M_Z)$ depend on scalar masses logarithmically. The quadratic dependence thus comes solely from the function $\Pi_{WW}(0) - \Pi_{WW}(M_W)$. When there is a large hierarchy among the three scalar masses (case (c) in Appendix B and its generalization), all contributions are of the same sign, and are proportional to the scalar mass squared,

$$\begin{aligned} \Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi \hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ \left. \left. + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] \right. \\ \left. - s_\delta^2 \frac{M_{H^0}^2 M_{H^\pm}^2}{2M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) - c_\delta^2 \frac{2M_{K^0}^2 M_{H^\pm}^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{K^0}^2} \right) \right\}, \end{aligned} \quad (31)$$

for $M_{H^0} \ll M_{K^0} \ll M_{H^\pm}$. Thus the scalar contribution to $\Delta r_{triplet}$ in this case is very large, and it grows with the scalar masses. On the other hand, when the mass splitting between either pair of the three scalar masses is small (case (a) and (b) and their generalization), the scalar contributions grow with the mass splitting^{10,27,28},

$$\begin{aligned} \Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi \hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ \left. \left. + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] \right. \\ \left. + \frac{5}{72} \left[s_\delta^2 \frac{(M_{H^\pm}^2 - M_{H^0}^2)}{M_{H^0}^2} + 4c_\delta^2 \frac{(M_{H^\pm}^2 - M_{K^0}^2)}{M_{K^0}^2} \right] \right\}, \end{aligned} \quad (32)$$

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for $M_{H^0} \simeq M_{K^0} \simeq M_{H^\pm}$, and,

$$\Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] - s_\delta^2 \frac{M_{H^0}^2 M_{H^\pm}^2}{2M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) + \frac{5}{18} c_\delta^2 \frac{(M_{H^\pm}^2 - M_{K^0}^2)}{M_{K^0}^2} \right\}, \quad (33)$$

for $M_{H^0} \ll M_{K^0} \simeq M_{H^\pm}$. Cancellations can occur in this case among contributions from different scalar fields, leading to the viability of a heavier neutral Higgs boson than is allowed in the SM²⁹.

The non-decoupling property of the triplet is seen in Eq. (31), (32) and (33). Because $\Delta r_{triplet}$ depends quadratically on the scalar masses^{6,30,31,32,33}, the scalars must be included in any effective field theory analysis of low energy physics.

The scalar potential of the model with a $SU(2)_L$ triplet and an $SU(2)_L$ doublet is given by the following²⁹:

$$V(H, \Phi) = \mu_1^2 |H|^2 + \frac{1}{2} \mu_2^2 \Phi^\dagger \Phi + \lambda_1 |H|^4 + \frac{1}{4} \lambda_2 |\Phi^\dagger \Phi|^2 + \frac{1}{2} \lambda_3 |H|^2 \Phi^\dagger \Phi + \lambda_4 \Phi_U^\alpha H^\dagger \sigma^\alpha H, \quad (34)$$

where σ^α denotes the Pauli matrices, and

$$\Phi_U = U^\dagger \Phi, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \\ -1 & -i & 0 \end{pmatrix}. \quad (35)$$

From the minimization conditions (see Appendix D),

$$\left. \frac{\partial V}{\partial \eta^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = \left. \frac{\partial V}{\partial \phi^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = 0, \quad (36)$$

we obtain the following conditions,

$$4\mu_2^2 t_\delta + \lambda_2 v^2 t_\delta^3 + 2\lambda_3 v^2 t_\delta - 4\lambda_4 v = 0 \quad (37)$$

$$\mu_1^2 + \lambda_1 v^2 + \frac{1}{8} \lambda_3 v^2 t_\delta^2 - \frac{1}{2} \lambda_4 v t_\delta = 0. \quad (38)$$

The two mixing angles, γ and δ , in the neutral and charged Higgs sectors defined in Eqs. (18) and (19), are solutions to the following two equations²⁹,

$$0 = \lambda_4 v + \tan \delta \left[\mu_1^2 - \mu_2^2 + \lambda_1 v^2 - \frac{1}{2} \lambda_3 v^2 + \lambda_4 v' - \lambda_2 v' + \frac{1}{2} \lambda_3 v' - \lambda_4 v \tan \delta \right] \quad (39)$$

$$0 = -\lambda_4 v + \lambda_3 v v' + \tan \gamma \left[\mu_1^2 - \mu_2^2 + 3\lambda_1 v^2 - \frac{1}{2} \lambda_3 v^2 - \lambda_4 v' - 3\lambda_2 v'^2 + \frac{1}{2} \lambda_3 v'^2 + \lambda_4 v \tan \gamma - \lambda_3 v v' \tan \gamma \right], \quad (40)$$

which are obtained by minimizing the scalar potential. In terms of the parameters in the scalar potential, the masses of the four scalar fields are given by²⁹,

$$M_{H^\pm}^2 = \mu_2^2 + \lambda_2 v^2 \tan^2 \delta + \lambda_4 v \tan \delta + \frac{1}{2} \lambda_3 v^2 \quad (41)$$

$$M_{H^0}^2 = \mu_1^2 + 3\lambda_1 v^2 + \lambda_3 v^2 \tan \delta \left(\frac{1}{2} \tan \delta - \tan \gamma \right) + \lambda_4 v \left(\tan \gamma - \tan \delta \right) \quad (42)$$

$$M_{K^0}^2 = \mu_2^2 + 3\lambda_2 v^2 \tan^2 \delta - \lambda_4 v \tan \gamma + \frac{1}{2} \lambda_3 v^2 \left(1 + 2 \tan \delta \tan \gamma \right). \quad (43)$$

This model has six parameters in the scalar sector, $(\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$. Equivalently, we can choose $(M_{H^0}, M_{K^0}, M_{H^\pm}, v, \tan \delta, \tan \gamma)$ as the independent parameters. Two of these six parameters, v and $\tan \delta$, contribute to the gauge boson masses.

When turning off the couplings between the doublet and the triplet in the scalar potential, $\lambda_3 = \lambda_4 = 0$, the triplet could still acquire a VEV, $v' \sim \sqrt{\frac{-2\mu_2^2}{\lambda_2}}$, provided that μ_2^2 is negative. Since we have not observed any light scalar experimentally up to the EW scale, the triplet mass which is roughly of order μ_2 has to be at least of the EW scale, $v \lesssim \mu_2$. This is problematic because the VEV of a real triplet only contributes to M_W but not to M_Z , which then results in a contribution of order $\mathcal{O}(1)$ to the ρ parameter, due to the relation, $\rho = 1 + 4\frac{v'^2}{v^2}$. For μ_2 greater than v , the EW symmetry is broken at a high scale. In order to avoid these problems, the parameter μ_2^2 thus has to be positive so that the triplet does not acquire a VEV via this mass term when λ_4 is turned off. Once the coupling λ_3 is turned on while keeping $\lambda_4 = 0$, the term $\lambda_3 |H|^2 \Phi^\dagger \Phi$ effectively plays the role of the mass term for Φ and for H . Thus, similar to the reasoning given above, for $\mu_2 \sim v$, the coupling λ_3 has to be positive so that it does not induce a large triplet VEV. For simplicity, consider the case when there is no mixing in the neutral Higgs sector, $\gamma = 0$. In this case, when the mixing in the charged sector approaches zero, $\delta \rightarrow 0$, the masses M_{K^0} and M_{H^\pm} approach infinity, and their difference $M_{K^0}^2 - M_{H^\pm}^2$ approaches zero. The contribution due to the new scalars thus vanishes, and only the lightest neutral Higgs contributes to $\Delta r_{\text{triplet}}$. Even though the contribution due to the new scalars vanishes, $\Delta r_{\text{triplet}}$ does not approach Δr_{SM} . This is because in the TM case, four input parameters are needed, while in the SM case three inputs are needed. There is no continuous limit that takes one case to the other^{5,6,30,31,32,33}.

One way to achieve the $\delta \rightarrow 0$ limit is to take the mass parameter $\mu_2^2 \rightarrow \infty$ while keeping the parameter λ_4 finite. Eq. (40) then dictates that $\mu_2^2 \tan \delta \sim \lambda_4 v \sim v^2$. However, satisfying Eq. (41) requires that $\lambda_4 v \sim \lambda_3 v v' = \lambda_3 v^2 \tan \beta$. As $\lambda_4 v \sim v^2$, this condition implies that the dimensionless coupling constant λ_3 has to scale as μ_2^2/v^2 , which approaches infinity as $\delta \rightarrow 0$. This can also be seen from Eq. (D.7). As there is no mixing in the neutral sector,

$$\frac{\partial^2 V}{\partial \phi^0 \partial \eta^0} = \frac{1}{2} \lambda_3 v^2 t_\delta - \lambda_4 v = 0, \quad (44)$$

the condition

$$\tan \delta = \frac{2\lambda_4}{\lambda_3 v} \quad (45)$$

then follows. So, in the absence of the neutral mixing, $\gamma = 0$, in order to take the charged mixing angle δ to zero while holding λ_4 fixed, one has to take λ_3 to infinity. In other words, for the triplet to decouple requires a dimensionless coupling constant λ_3 to become strong, leading to the breakdown of the perturbation theory.

Alternatively, the neutral mixing angle γ can approach zero by taking $\mu_2^2 \rightarrow \infty$ while keeping λ_3 and λ_4 fixed. In this case, the minimization condition,

$$4\mu_2^2 t_\delta + \lambda_2 v^2 t_\delta^3 + 2\lambda_3 v^2 t_\delta - 4\lambda_4 v = 0, \quad (46)$$

where $t_\delta \equiv \tan \delta$, implies that the charged mixing angle δ has to approach zero. This again corresponds to the case where the custodial symmetry is restored, by which we mean that the triplet VEV vanishes, $v' = 0$. In this case, severe fine-tuning is needed in order to satisfied the condition given in Eq. (41). Another way to get $\delta \rightarrow 0$ is to have $\lambda_4 \rightarrow 0$, which trivially satisfies Eq. (40). This can also be seen from Eq. D.4,

$$\frac{\partial^2 V}{\partial \eta^+ \partial \phi^-} = \lambda_4 v = 0. \quad (47)$$

Eq. (41) then gives $\lambda_4 \cot \delta \sim \lambda_3 v$. So for small λ_3 , the masses of these additional scalar fields are of the weak scale, $M_{K^0} \sim M_{H^\pm} \sim v$. This corresponds to a case when the custodial symmetry is restored. So unless one imposes by hand such symmetry to forbid λ_4 , four input parameters are always needed in the renormalization. If there is a symmetry which makes $\lambda_4 = 0$ (to all orders), say, $\Phi \rightarrow -\Phi$, then there are only three input parameters needed. So the existence of such a symmetry is crucial when one-loop radiative corrections are concerned.

5. Results

The previous section has presented analytic results for the triplet model, demonstrating that the dependence of the W mass on the top quark mass is logarithmic, while the dependence on the scalar masses is quadratic. A dramatic change in the behavior of the W mass is also observed in the $SU(2)_L \times SU(2)_R$ model^{34,35,36}. For comparison with the triplet model, we summarize the results of the left-right model in Appendix Appendix C. In this case, the dependence of the W mass on the top quark mass is weakened from that of the SM since it depends on $m_t^2/M_{W_2}^2$, where M_{W_2} is the heavy charged gauge boson mass of the left-right model.

The dependence of the W mass on the top quark mass, m_t , in the case of the SM, the model with a triplet Higgs, and the minimal left-right model, are shown in Fig. 1. For the SM, we include the complete contributions from top and bottom quarks, the SM Higgs boson with $M_{H^0} = 120$ GeV, and the gauge bosons. In this case, the m_t dependence in the prediction for M_W is quadratic. The range of values

for the input parameter m_t that give a prediction for M_W consistent with the experimental 1σ limits¹, $M_W = 80.425 \pm 0.0666$ GeV, is very narrow. It coincides with the current experimental limits¹, 178 ± 4.3 GeV. For the triplet model and the LR model, we include only the top quark contribution. As we have shown in Sec. 3, the prediction for M_W in the triplet model depends on m_t only logarithmically. In our numerical result for the left-right model, we have used $(G_\mu, \alpha(M_Z), \hat{s}_\theta, M_Z, M_{W_2})$ in the gauge sector, in addition to m_t in the fermion sector, to predict M_W . Here we have identified W_1 and Z_1 as the W and Z bosons in the SM and consequently $M_W = M_{W_1}$ and $M_Z = M_{Z_1}$. In this case, the m_t dependence in the prediction

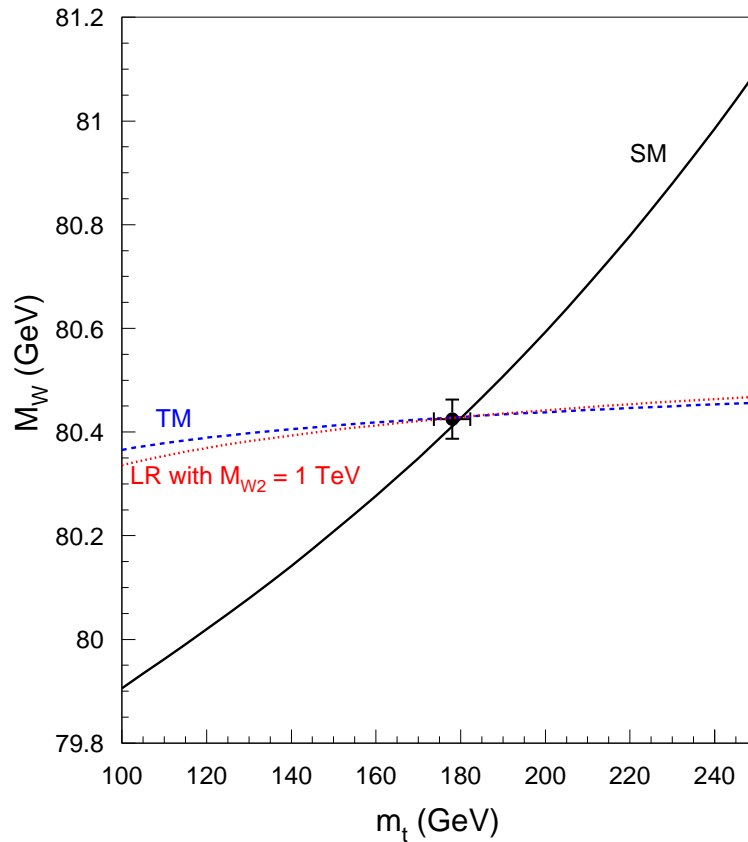


Fig. 1. Prediction for the W mass as a function of the top quark mass in the SM, TM and the LR model. The data point represents the experimental values with 1σ error bars¹. For the SM, we include the complete contributions from top and bottom quarks, the SM Higgs boson with $M_{H^0} = 120$ GeV, and the gauge bosons. For the TM and the LR model, we include only the top quark contribution and the absolute normalization is fixed so that the curves intersect the data point. The W_2 boson mass is chosen to be $M_{W_2} = 1$ TeV in the LR model.

for M_W is similarly softer because the top quark contributions are suppressed by a heavy scale, M_{W_2} . In the triplet model and the left-right model, the range of m_t that gives a prediction for M_W consistent with the experimental value is thus much larger, ranging from $m_t = 120$ to 250 GeV. The presence of the triplet Higgs thus dramatically changes the m_t dependence in M_W . This is clearly demonstrated in Fig. 1 by the almost flat curves of the triplet and left-right symmetric models, contrary to that of the SM, which is very sensitive to m_t . In Fig. 2, we show the prediction for M_W as a function of m_t in the triplet model, with $\alpha(M_Z)$ and \hat{s}_θ varying within the 1σ limits^{4,1}, $\alpha(M_Z)^{-1} = 128.91 \pm 0.0392$ and $\hat{s}_\theta^2 = 0.2315 \pm 0.000314$. We find that the prediction for M_W is very sensitive to the input parameters $\alpha(M_Z)$ and \hat{s}_θ .

The complete contributions from the top and bottom quarks and the SM gauge bosons, as well as all four scalar fields in the triplet model are included in Fig. 3 and 4. We have also included the box and vertex corrections. In Fig. 3, we show the prediction in the triplet model for M_W as a function of m_t , allowing M_{H^0} , M_{H^\pm} and M_{K^0} to vary independently between 1 – 3 TeV, 300 – 600 GeV and 500 – 600 GeV. Interestingly, for all scalar masses in the range of 1 – 3 TeV, the prediction for

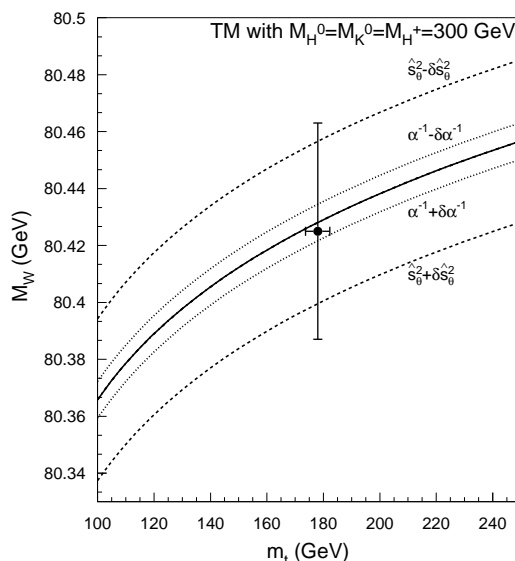


Fig. 2. Prediction for the W mass in the TM as a function of the top quark mass for scalar masses $M_{H^0} = M_{K^0} = M_{H^\pm} = 300$ GeV, with $\alpha(M_Z)$ and \hat{s}_θ varying within their 1σ limits^{4,1} $\alpha(M_Z)^{-1} = 128.91 \pm 0.0392$ and $\hat{s}_\theta^2 = 0.2315 \pm 0.000314$. The solid curve indicates the prediction with $\alpha(M_Z)$ and \hat{s}_θ taking the experimental central values, $\alpha(M_Z)^{-1} = 128.91$ and $\hat{s}_\theta^2 = 0.2315$. The area bounded by the short dashed (dotted) curves indicates the prediction with $\alpha(M_Z)^{-1} = 128.91$ ($\hat{s}_\theta^2 = 0.2315$) and \hat{s}_θ^2 ($\alpha(M_Z)$) varying with its 1σ limits. The data point represents the experimental values with 1σ error bars¹.

M_W in the TM model still agrees with the experimental 1σ limits. Fig. 4 shows the prediction for M_W as a function of M_{H^0} for various values of M_{K^0} and M_{H^\pm} . For small $M_{K^0}^2 - M_{H^\pm}^2$, the lightest neutral Higgs boson mass can range from $M_{H^0} = 100$ GeV to a TeV and still satisfy the experimental prediction for M_W . This agrees with our conclusion in Sec. 3 that to minimize the scalar contribution to $\Delta r_{triplet}$, the mass splitting $M_{K^0}^2 - M_{H^\pm}^2$ has to be small and that when the mass splitting is small, cancellations can occur between the contributions of the lightest neutral Higgs and those of the additional scalar fields. This has new important implications for the Higgs searches.

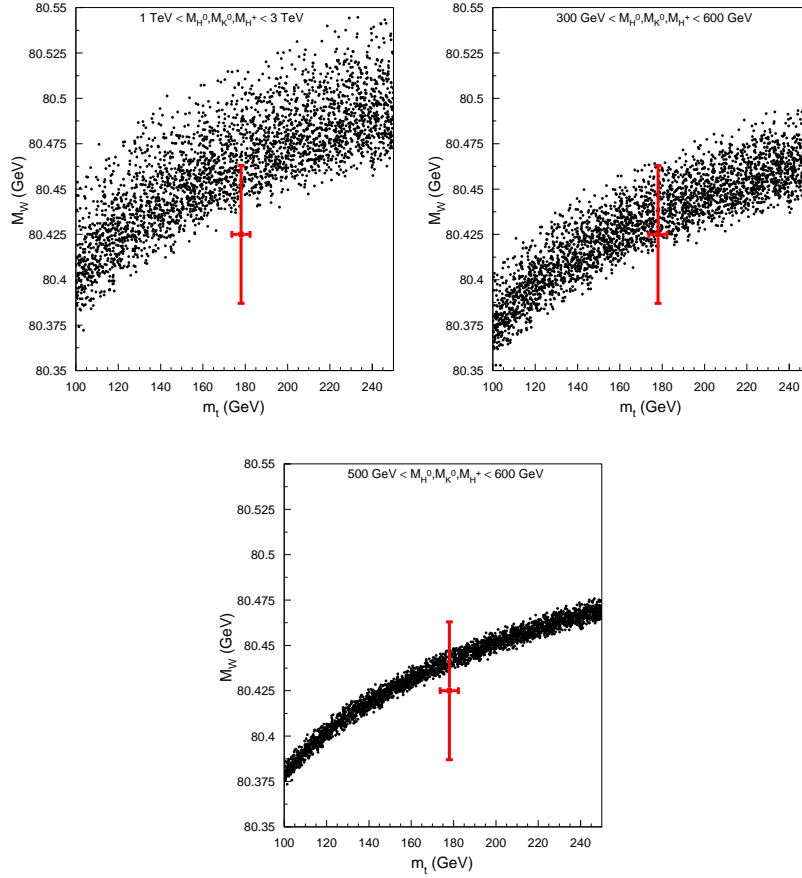


Fig. 3. Prediction for the W mass in the TM as a function of the top quark mass for scalar masses, M_{H^0} , M_{K^0} and M_{H^\pm} , varying independently between (a) 1 – 3 TeV, (b) 300 – 600 GeV, and (c) 500 – 600 GeV. The data point represents the experimental values with 1σ error bars¹.

6. Conclusion

We have considered the top quark contribution to muon decay at one loop in the SM and in two models with $\rho \neq 1$ at tree level: the SM with an addition real scalar triplet and the minimal left-right model. In these new models, because the ρ parameter is no longer equal to one at the tree level, a fourth input parameter is required in a consistent renormalization scheme. These models illustrate a general feature that the m_t dependence in the radiative corrections $\Delta r_{triplet}$ becomes logarithmic, contrary to the case of the SM where Δr_{SM} depends on m_t quadratically. One therefore loses the prediction for m_t from radiative corrections. On the other hand, due to cancellations between the contributions to the radiative corrections from the SM Higgs and the triplet, a Higgs mass M_{H^0} as large as a few TeV is allowed by

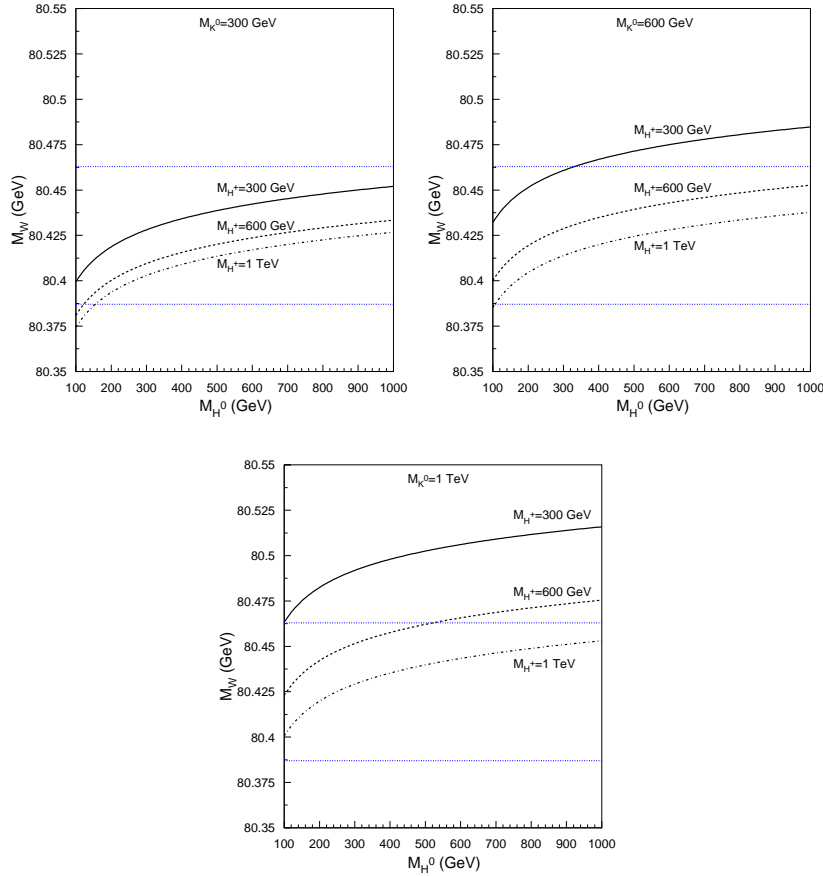


Fig. 4. Prediction for the W mass in the TM as a function of the lightest neutral Higgs boson mass, M_{H^0} , for various values of M_{K^0} and M_{H^\pm} . The area bounded by the two horizontal lines is the 1σ allowed region for M_W ¹.

the W mass measurement. We emphasize that by taking the triplet mass to infinity, one does not recover the SM. This is due to the fact that the triplet scalar field is non-decoupling, and it implies that the one-loop electroweak results cannot be split into a SM contribution plus a piece which vanishes as the scale of new physics becomes much larger than the weak scale. This fact has been overlooked by most analyses in the littlest Higgs model^{37,38}, and correctly including the effects of the triplet can dramatically change the conclusion on the viability of the model^{10,11}. Such non-decoupling effect has been pointed out in the two Higgs doublet model²⁷, left-right symmetric model²⁸, and the littlest Higgs model¹⁰. It has *not* been discussed before in the model with a triplet. We comment that the non-decoupling observed in these examples do not contradict with the common knowledge that in GUT models heavy scalars decouple. These two cases are fundamentally different because in GUT models, heavy scalar fields do not acquire VEV that break the EW symmetry, while in cases where non-decoupling is observed, heavy scalar fields *do* acquire VEV that breaks the EW symmetry. The quadratic dependence in scalar masses in the triplet model can be easily understood physically. In SM with only the Higgs doublet present, the quadratic scalar mass contribution is protected by the tree level custodial symmetry, and thus the Higgs mass contribution is logarithmic at one-loop. This is the well-known screening theorem by Veltman³⁹. As the custodial symmetry is broken in the SM at one-loop due to the mass splitting between the top and bottom quarks, the two-loop Higgs contribution is quadratic. In models with a triplet Higgs, as the custodial symmetry is broken already at the tree level, there is no screening theorem that protects the quadratic scalar mass dependence from appearing. Our results demonstrate the importance of performing the renormalization correctly according to the EW structure of the new models.

7. Acknowledgments

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Appendix A. Contributions of the top loop

We summarize below the leading contributions due to the SM top loop to the self-energies of the gauge bosons¹⁰, where the definitions of the Passarino-Veltman

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functions utilized below are given in¹⁰.

$$\begin{aligned} \Pi^{WW}(M_W^2) = & -\frac{3\alpha}{4\pi s_\theta^2} \left[A_0(m_t^2) + m_b^2 B_0(M_W^2, m_b^2, m_t^2) \right. \\ & \left. - M_W^2 B_1(M_W^2, m_b^2, m_t^2) - 2B_{22}(M_W^2, m_b^2, m_t^2) \right] \end{aligned} \quad (\text{A.1})$$

$$\Pi^{WW}(0) = -\frac{3\alpha}{16\pi s_\theta^2} \cdot m_t^2 \left[1 + 2 \ln \left(\frac{Q^2}{m_t^2} \right) \right] \quad (\text{A.2})$$

$$\begin{aligned} \Pi^{ZZ}(M_Z^2) = & -\frac{3\alpha}{8\pi s_\theta^2 c_\theta^2} \left[\left(\left(\frac{1}{2} - \frac{4}{3} s_\theta^2 \right)^2 + \frac{1}{4} \right) h_1(m_t^2) \right. \\ & \left. - \frac{8}{3} s_\theta^2 \left(1 - \frac{4}{3} s_\theta^2 \right) h_2(m_t^2) \right] \end{aligned} \quad (\text{A.3})$$

$$\Pi'_{\gamma\gamma}(0) = \frac{4\alpha}{9\pi} \ln \left(\frac{Q^2}{m_t^2} \right) \quad (\text{A.4})$$

$$\Pi^{\gamma Z}(M_Z^2) = -\frac{\alpha}{\pi s_\theta c_\theta} \left(\frac{1}{2} - \frac{4}{3} s_\theta^2 \right) M_Z^2 \left[\frac{1}{3} \ln \left(\frac{Q^2}{m_t^2} \right) - 2I_3 \left(\frac{M_Z^2}{m_t^2} \right) \right] \quad (\text{A.5})$$

where

$$h_1(m_t^2) = 2m_t^2 \left[\ln \left(\frac{Q^2}{m_t^2} \right) - \frac{1}{3} I_1 \left(\frac{M_Z^2}{m_t^2} \right) \right] \quad (\text{A.6})$$

$$h_2(m_t^2) = m_t^2 \left[I_1 \left(\frac{M_Z^2}{m_t^2} \right) - \ln \left(\frac{Q^2}{m_t^2} \right) \right] \quad (\text{A.7})$$

The integrals are defined as,

$$\begin{aligned} I_1(a) &= \int_0^1 dx \ln \left(1 - ax(1-x) \right) \\ I_3(a) &= \int_0^1 dx x(1-x) \ln \left(1 - ax(1-x) \right). \end{aligned}$$

Here s_θ is defined in the on-shell scheme (Eq. (8)) for the SM and as the effective weak mixing angle (Eq. (23)) for the TM and LR model.

Appendix B. Contributions of the scalars in a model with a triplet Higgs

The complete contributions to various two-point functions that appear in $\Delta r_{\text{triplet}}$ are given below, where the scalar and fermion contributions are given in Ref. 25,

10, and we have taken the SM gauge boson contributions from Ref. 18.

$$\begin{aligned}
\Pi_{WW}(0) - \Pi_{WW}(M_W) = & -\frac{3\alpha}{16\pi\hat{s}_\theta^2} \cdot m_t^2 \left[1 + 2 \ln \left(\frac{Q^2}{m_t^2} \right) \right] \\
& + \frac{3\alpha}{4\pi\hat{s}_\theta^2} \left[A_0(m_t^2) + m_b^2 B_0(M_W^2, m_b^2, m_t^2) \right. \\
& \quad \left. - M_W^2 B_1(M_W^2, m_b^2, m_t^2) - 2B_{22}(M_W^2, m_b^2, m_t^2) \right] \\
& + \frac{\alpha}{4\pi\hat{s}_\theta^2} \left\{ s_\delta^2 H(M_{H^0}, M_{H^\pm}) + c_\delta^2 H(M_{H^0}, M_W) \right. \\
& \quad + 4c_\delta^2 H(M_{K^0}, M_{H^\pm}) + 4s_\delta^2 H(M_{K^0}, M_W) \\
& \quad \left. + s_\delta^2 H(M_Z, M_{H^\pm}) + c_\delta^2 H(M_Z, M_W) \right\} \\
& + \frac{\alpha}{4\pi\hat{s}_\theta^2} M_W^2 \left[\frac{s_\delta^2 c_\delta^2}{\hat{c}_\theta^2} \left(B_0(0, M_Z, M_{H^\pm}) - B_0(M_W, M_Z, M_{H^\pm}) \right) \right. \\
& \quad + \frac{(s_\delta^2 - \hat{s}_\theta^2)^2}{\hat{c}_\theta^2} \left(B_0(0, M_Z, M_W) - B_0(M_W, M_Z, M_W) \right) \\
& \quad + \hat{s}_\theta^2 \left(B_0(0, 0, M_W) - B_0(M_W, 0, M_W) \right) \\
& \quad + c_\delta^2 \left(B_0(0, M_{H^0}, M_W) - B_0(M_W, M_{H^0}, M_W) \right) \\
& \quad \left. + 4s_\delta^2 \left(B_0(0, M_{K^0}, M_W) - B_0(M_W, M_{K^0}, M_W) \right) \right] \\
& + \frac{\alpha}{4\pi\hat{s}_\theta^2} \left[\hat{c}_\theta^2 \left(A_1(0, M_Z, M_W) - A_1(M_W, M_Z, M_W) \right) \right. \\
& \quad + \hat{s}_\theta^2 \left(A_1(0, 0, M_W) - A_1(M_W, 0, M_W) \right) \\
& \quad \left. - 2\hat{c}_\theta^2 H(M_Z, M_W) - 2\hat{s}_\theta^2 H(0, M_W) \right],
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
\Pi_{\gamma Z}(M_Z) = & -\frac{\alpha}{\pi \hat{s}_\theta \hat{c}_\theta} \left(\frac{1}{2} - \frac{4}{3} \hat{s}_\theta^2 \right) M_Z^2 \left[\frac{1}{3} \ln \left(\frac{Q^2}{m_t^2} \right) - 2I_3 \left(\frac{M_Z^2}{m_t^2} \right) \right] \\
& + \frac{\alpha}{4\pi \hat{s}_\theta \hat{c}_\theta} \left[2(c_\delta^2 - \hat{s}_\theta^2 + \hat{c}_\theta^2) B_{22}(M_Z, M_{H^\pm}, M_{H^\pm}) \right. \\
& \quad + 2(s_\delta^2 - \hat{s}_\theta^2 + \hat{c}_\theta^2) B_{22}(M_Z, M_W, M_W) \\
& \quad \left. + (\hat{s}_\theta^2 - \hat{c}_\theta^2 - c_\delta^2) A(M_{H^\pm}) + (\hat{s}_\theta^2 - \hat{c}_\theta^2 - s_\delta^2) A(M_W) \right] \\
& + \frac{\alpha}{4\pi} (2M_W^2) \frac{\hat{s}_\theta^2 - s_\delta^2}{\hat{s}_\theta \hat{c}_\theta} B_0(M_Z, M_W, M_W) \\
& - \frac{\alpha}{4\pi \hat{s}_\theta^2} \left[\hat{s}_\theta \hat{c}_\theta A_1(M_Z, M_W, M_W) + 2\hat{c}_\theta \hat{s}_\theta A_2(M_W) \right. \\
& \quad \left. + 2\hat{s}_\theta \hat{c}_\theta B_{22}(M_Z, M_W, M_W) \right], \tag{B.2}
\end{aligned}$$

$$\Pi'_{\gamma\gamma}(0) = \frac{\alpha}{\pi} \left[\frac{4}{9} \ln \left(\frac{Q^2}{m_t^2} \right) + \frac{1}{12} \ln \left(\frac{Q^2}{M_{H^\pm}^2} \right) - \frac{3}{4} \ln \left(\frac{Q^2}{M_W^2} \right) - \frac{1}{6} \right], \tag{B.3}$$

$$\begin{aligned}
\Pi_{\gamma Z}(0) = & \frac{\alpha}{4\pi} \left[\frac{(\hat{s}_\theta^2 - s_\delta^2)}{\hat{s}_\theta \hat{c}_\theta} 2M_W^2 B_0(0, M_W, M_W) - \frac{\hat{c}_\theta}{\hat{s}_\theta} A_1(0, M_W, M_W) \right. \\
& \left. - 2\frac{\hat{c}_\theta}{\hat{s}_\theta} A_2(M_W) - 2\frac{\hat{c}_\theta}{\hat{s}_\theta} B_{22}(0, M_W, M_W) \right], \tag{B.4}
\end{aligned}$$

where

$$H(m_1, m_2) = -B_{22}(0, m_1, m_2) + B_{22}(M_W, m_2, m_2), \tag{B.5}$$

$$A_1(p, m_1, m_2) = -A_0(m_1) - A_0(m_2) - (m_1^2 + m_2^2 + 4p^2) B_0(p, m_1, m_2) \tag{B.6}$$

$$-10B_{22}(p, m_1, m_2) + 2(m_1^2 + m_2^2 - \frac{p^2}{3}),$$

$$A_2(m) = 3A_0(m) - 2m^2, \tag{B.7}$$

and \bar{s}_θ is defined in Eq. (23).

To extract the dependence on the masses of the lightest neutral Higgs, M_{H^0} , and the extra scalar fields, M_{K^0} and M_{H^\pm} , we first note that, in the limit $\delta m^2 \ll m_1^2$,

$$B_0(p, m_1, m_2) = \ln \left(\frac{Q^2}{m_1^2} \right) + \frac{1}{6} \frac{p^2}{m_1^2} - \frac{1}{2} \frac{\delta m^2}{m_1^2} + \mathcal{O} \left((\delta m^2)^2, \left(\frac{p^2}{m_1^2} \right)^2 \right) \tag{B.8}$$

$$\begin{aligned}
B_{22}(p, m_1, m_2) = & \frac{1}{2} m_1^2 \left[1 + \ln \left(\frac{Q^2}{m_1^2} \right) \right] - \frac{1}{12} p^2 \ln \left(\frac{Q^2}{m_1^2} \right) - \frac{1}{72} \frac{p^4}{m_1^2} \\
& + \left[\frac{1}{4} \ln \left(\frac{Q^2}{m_1^2} \right) + \frac{5}{72} \frac{p^2}{m_1^2} \right] \delta m^2 + \mathcal{O} \left((\delta m^2)^2, \left(\frac{p^2}{m_1^2} \right)^2 \right) \\
& + (\text{terms with no scalar dependence}),
\end{aligned} \tag{B.9}$$

where we have defined $\delta m^2 = m_2^2 - m_1^2$ and assumed that $p^2 \ll m_1^2$. Using these relations, we then have,

$$H(m_1, m_2) = \frac{5}{72} \frac{M_W^2}{m_1^2} \delta m^2 - \frac{1}{72} \frac{M_W^4}{m_1^2} + \mathcal{O}\left((\delta m^2)^2, \left(\frac{M_W^2}{m_1^2}\right)^2\right) + (\text{terms with no scalar dependence}), \quad (\text{B.10})$$

$$B_0(0, m_1, m_2) - B_0(M_W, m_1, m_2) = -\frac{1}{6} \frac{M_W^2}{m_1^2} + \mathcal{O}\left((\delta m^2)^2, \left(\frac{M_W^2}{m_1^2}\right)^2\right) \quad (\text{B.11})$$

On the other hand, in the limit $m_1 \gg m_2$, we get,

$$B_0(p, m_1, m_2) = \left(1 + \ln\left(\frac{Q^2}{m_1^2}\right)\right) \left(1 + \frac{m_2^2}{m_1^2}\right) + \frac{1}{2} \frac{p^2}{m_1^2} + \mathcal{O}\left(\left(\frac{m_2^2}{m_1^2}\right)^2, \left(\frac{p^2}{m_1^2}\right)^2\right) \quad (\text{B.12})$$

$$\begin{aligned} B_{22}(p, m_1, m_2) = & m_1^2 \left(\frac{3}{8} + \frac{2}{3} \frac{m_2^2}{m_1^2} - \frac{1}{18} \frac{p^2}{m_1^2} - \frac{m_2^2}{12p^2} \right) \\ & - \left(\frac{1}{4} + \frac{1}{6} \frac{m_2^2}{m_1^2} + \frac{m_2^2}{12p^2} - \frac{p^2}{12m_1^2} \right) m_1^2 \ln m_1^2 \\ & - \left(\frac{m_2^2}{12m_1^2} - \frac{m_2^2}{12p^2} \right) m_1^2 \ln m_2^2 + \left(\frac{1}{4} m_1^2 + \frac{1}{4} m_2^2 - \frac{p^2}{12} \right) \ln Q^2 \\ & + \mathcal{O}\left(\left(\frac{m_2^2}{m_1^2}\right), \left(\frac{p^2}{m_1^2}\right)\right) + (\text{terms with no scalar dependence}), \end{aligned} \quad (\text{B.13})$$

which gives,

$$H(m_1, m_2) = -\frac{m_1^2 m_2^2}{12 M_W^2} \left[1 + \ln\left(\frac{m_1^2}{m_2^2}\right) \right] + \mathcal{O}\left(\left(\frac{m_2^2}{m_1^2}\right), \left(\frac{p^2}{m_1^2}\right)\right) + (\text{terms with no scalar dependence}) \quad (\text{B.14})$$

$$B_0(0, m_1, m_2) - B_0(M_W, m_1, m_2) = -\frac{1}{2} \frac{M_W^2}{m_1^2} + \mathcal{O}\left(\left(\frac{m_2^2}{m_1^2}\right)^2, \left(\frac{M_W^2}{m_1^2}\right)^2\right) \quad (\text{B.15})$$

The two-point function $\Pi_{\gamma Z}(0)$ does not have any scalar dependence, and the function $\Pi'_{\gamma\gamma}(0)$ depends on the scalar mass only logarithmically,

$$\Pi'_{\gamma\gamma}(0) \rightarrow \frac{\alpha}{12\pi} \ln\left(\frac{Q^2}{M_{H^\pm}^2}\right). \quad (\text{B.16})$$

The scalar dependence in the function $\Pi_{\gamma Z}(M_Z)$ is,

$$\begin{aligned} \Pi_{\gamma Z}(M_Z) \rightarrow & \frac{\alpha}{4\pi \hat{s}_\theta \hat{c}_\theta} (\hat{s}_\theta^2 - \hat{c}_\theta^2 - c_\delta^2) \left[A_0(M_{H^\pm}) - 2B_{22}(M_Z, M_{H^\pm}, M_{H^\pm}) \right] \\ & = \frac{\alpha}{4\pi \hat{s}_\theta \hat{c}_\theta} (\hat{s}_\theta^2 - \hat{c}_\theta^2 - c_\delta^2) M_Z^2 \left[\frac{1}{12} \ln\left(\frac{M_{H^\pm}^2}{Q^2}\right) - \frac{1}{72} \frac{M_Z^2}{M_{H^\pm}^2} \right], \end{aligned} \quad (\text{B.17})$$

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thus the dependence is also logarithmic. On the other hand, in the function $\Pi_{WW}(0) - \Pi_{WW}(M_W)$, the scalar dependence is given by,

$$\begin{aligned} \Pi_{WW}(0) - \Pi_{WW}(M_W) \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} & \left\{ s_\delta^2 H(M_{H^0}, M_{H^\pm}) + c_\delta^2 H(M_{H^0}, M_W) \right. \\ & + 4c_\delta^2 H(M_{K^0}, M_{H^\pm}) + 4s_\delta^2 H(M_{K^0}, M_W) + s_\delta^2 H(M_Z, M_{H^\pm}) \Big\} \\ & + \frac{\alpha}{4\pi\hat{s}_\theta^2} M_W^2 \left[\frac{s_\delta^2 c_\delta^2}{\hat{c}_\theta^2} \left(B_0(0, M_Z, M_{H^\pm}) - B_0(M_W, M_Z, M_{H^\pm}) \right) \right. \\ & + c_\delta^2 \left(B_0(0, M_{H^0}, M_W) - B_0(M_W, M_{H^0}, M_W) \right) \\ & \left. + 4s_\delta^2 \left(B_0(0, M_{K^0}, M_W) - B_0(M_W, M_{K^0}, M_W) \right) \right]. \end{aligned} \quad (\text{B.18})$$

From Eqs. (B.11) and (B.15), we know that the contributions from the terms in the square brackets of Eq. (B.18) are logarithmic. Thus the only possible quadratic dependence comes from terms in the curly brackets. We consider the following three limits, assuming that all scalar masses are much larger than M_W and M_Z :

- (a) $M_{H^0} \simeq M_{K^0} \simeq M_{H^\pm}$: In this case, the leading order scalar dependence is given by,

$$\begin{aligned} \Pi_{WW}(0) - \Pi_{WW}(M_W) \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} & \left\{ -\frac{1}{2} \left[c_\delta^2 M_{H^0}^2 \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ & + 4s_\delta^2 M_{K^0}^2 \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \Big] \\ & \left. + \frac{5}{72} \left[s_\delta^2 \frac{M_W^2}{M_{H^0}^2} (M_{H^\pm}^2 - M_{H^0}^2) + 4c_\delta^2 \frac{M_W^2}{M_{K^0}^2} (M_{H^\pm}^2 - M_{K^0}^2) \right] \right\}. \end{aligned} \quad (\text{B.19})$$

So the dominant scalar contribution to $\Delta r_{triplet}^S$ in this case is given by,

$$\begin{aligned} \Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} & \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ & + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \Big] \\ & \left. + \frac{5}{72} \left[s_\delta^2 \frac{(M_{H^\pm}^2 - M_{H^0}^2)}{M_{H^0}^2} + 4c_\delta^2 \frac{(M_{H^\pm}^2 - M_{K^0}^2)}{M_{K^0}^2} \right] \right\}. \end{aligned} \quad (\text{B.20})$$

- (b) $M_{H^0} \ll M_{K^0} \simeq M_{H^\pm}$: In this limit, the leading scalar dependence becomes,

$$\begin{aligned} \Pi_{WW}(0) - \Pi_{WW}(M_W) \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} & \left\{ -\frac{1}{2} \left[c_\delta^2 M_{H^0}^2 \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ & + 4s_\delta^2 M_{K^0}^2 \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \Big] \\ & \left. - s_\delta^2 \frac{M_{H^0}^2}{2M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) M_{H^\pm}^2 + \frac{5}{18} c_\delta^2 \frac{M_W^2}{M_{K^0}^2} (M_{H^\pm}^2 - M_{K^0}^2) \right\}. \end{aligned} \quad (\text{B.21})$$

The leading scalar contribution to $\Delta r_{triplet}^S$ is,

$$\begin{aligned} \Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ \left. \left. + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] \right. \\ \left. - s_\delta^2 \frac{M_{H^0}^2 M_{H^\pm}^2}{2M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) + \frac{5}{18} c_\delta^2 \frac{(M_{H^\pm}^2 - M_{K^0}^2)}{M_{K^0}^2} \right\}. \end{aligned} \quad (\text{B.22})$$

(c) $M_{H^0} \ll M_{K^0} \ll M_{H^\pm}$: In this limit, the leading scalar dependence becomes,

$$\begin{aligned} \Pi_{WW}(0) - \Pi_{WW}(M_W) \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 M_{H^0}^2 \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ \left. \left. + 4s_\delta^2 M_{K^0}^2 \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] \right. \\ \left. - s_\delta^2 \frac{M_{H^0}^2}{2M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) M_{H^\pm}^2 - c_\delta^2 \frac{2M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{H^\pm}^2}{M_{K^0}^2} \right) M_{H^\pm}^2 \right\}, \end{aligned} \quad (\text{B.23})$$

The leading scalar contribution to $\Delta r_{triplet}^S$ is thus given by,

$$\begin{aligned} \Delta r_{triplet}^S \rightarrow \frac{\alpha}{4\pi\hat{s}_\theta^2} \left\{ -\frac{1}{2} \left[c_\delta^2 \frac{M_{H^0}^2}{M_W^2} \ln \left(\frac{M_{H^0}^2}{M_W^2} \right) \right. \right. \\ \left. \left. + 4s_\delta^2 \frac{M_{K^0}^2}{M_W^2} \ln \left(\frac{M_{K^0}^2}{M_W^2} \right) + s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_Z^2} \right) \right] \right. \\ \left. - s_\delta^2 \frac{M_{H^0}^2 M_{H^\pm}^2}{2M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{H^0}^2} \right) - c_\delta^2 \frac{2M_{K^0}^2 M_{H^\pm}^2}{M_W^4} \ln \left(\frac{M_{H^\pm}^2}{M_{K^0}^2} \right) \right\}. \end{aligned} \quad (\text{B.24})$$

For the case $M_{H^0} \ll M_{H^\pm} \ll M_{K^0}$, make the replacement, $\ln \left(\frac{M_{H^\pm}^2}{M_{K^0}^2} \right) \longleftrightarrow \ln \left(\frac{M_{K^0}^2}{M_{H^\pm}^2} \right)$.

Appendix C. The Left-Right Symmetric Model

As our second example to show that new physics does not decouple from the SM at one-loop, we consider the left-right (LR) symmetric model^{40,41,42,43} which is defined by the gauge group,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \quad (\text{C.1})$$

The minimal left-right symmetric model contains a scalar bi-doublet, Φ , and two $SU(2)$ triplets, Δ_L and Δ_R . We assume that the scalar potential is arranged such

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that the Higgs fields obtain the following VEV's:

$$\Phi \sim (1/2, 1/2, 0) \sim \begin{pmatrix} \kappa \\ \kappa' \end{pmatrix} \quad (\text{C.2})$$

$$\Delta_L \sim (1, 0, 2) \sim \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \quad (\text{C.3})$$

$$\Delta_R \sim (0, 1, 2) \sim \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad (\text{C.4})$$

where the quantum numbers of these Higgs fields under $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ are given inside the parentheses. The VEV v_R breaks the $SU(2)_R \times U(1)_{B-L}$ symmetry down to $U(1)_Y$ of the SM, while the bi-doublet VEV's κ and κ' break the electroweak symmetry; the VEV v_L may be relevant for generating neutrino masses^{44,45,46,47,48,49,50,51,52}. After the symmetry breaking, there are two charged gauge bosons, W_1 and W_2 , two heavy neutral gauge bosons, Z_1 and Z_2 , and the massless photon. We will assume that W_1 and Z_1 are the lighter gauge bosons and obtain roughly their SM values after the symmetry breaking.

Turning off the $SU(2)_L$ triplet VEV, $v_L = 0$, and assuming for simplicity that the $SU(2)_L \times SU(2)_R$ gauge coupling constants satisfy $g_L = g_R = g$, there are five fundamental parameters in the gauge/Higgs sector,

$$(g, g', \kappa, \kappa', v_R). \quad (\text{C.5})$$

We can equivalently choose

$$(\alpha, M_{W_1}^2, M_{W_2}^2, M_{Z_1}^2, M_{Z_2}^2) \quad (\text{C.6})$$

as input parameters. The counter term for the weak mixing angle is then defined through these parameters and their counter terms. Assuming that the heavy gauge bosons are much heavier than the SM gauge bosons, $M_{W_2}, M_{Z_2} \gg M_{W_1}, M_{Z_1}$, then to leading order $\mathcal{O}(M_{W_1}^2/M_{W_2}^2)$, the counterterm $\delta\hat{s}_\theta$ is given as follows^{34,35,36},

$$\begin{aligned} \frac{\delta\hat{s}_\theta^2}{\hat{s}_\theta^2} &= 2 \frac{\hat{c}_\theta^2}{\hat{s}_\theta^2} \frac{(\delta M_{Z_1}^2 + \delta M_{Z_2}^2) - (\delta M_{W_1}^2 + \delta M_{W_2}^2)}{(M_{Z_1}^2 + M_{Z_2}^2) - (M_{W_1}^2 + M_{W_2}^2)} + \mathcal{O}(M_{W_1}^2/M_{W_2}^2) \\ &\simeq -2 \frac{\hat{c}_\theta^2}{\hat{s}_\theta^2} (\hat{c}_\theta^2 - \hat{s}_\theta^2) \frac{\delta M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} + \dots \\ &\simeq \frac{\sqrt{2}G_\mu}{8\pi^2} \hat{c}_\theta^2 \left(\frac{\hat{c}_\theta^2}{\hat{s}_\theta^2} - 1 \right) \frac{M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} \cdot (3m_t^2), \end{aligned} \quad (\text{C.7})$$

where the effective weak mixing angle, \hat{s}_θ , is defined as in Eq. (23). To go from the first to the second step in the above equation, we have used the following relation, $(M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2) = \frac{g^2}{2\cos^2 2\theta_W} v_R^2 \sim \frac{1}{\cos^2 2\theta_W} (M_{W_2}^2 - M_{W_1}^2)$. In the third line of Eq. (C.7), we include only the leading top quark mass dependence.

When the limit $M_{W_2} \rightarrow \infty$ is taken, $\delta\hat{s}_\theta^2/\hat{s}_\theta^2$ approaches zero, and thus the SM result, $\delta\hat{s}_\theta^2/\hat{s}_\theta^2 \sim m_t^2$ is *not* recovered, which is not what one would naively expect.

One way to understand this is that in the left-right model, four input parameters are held fixed, while in the SM three input parameters are fixed. There is thus no continuous limit which takes one from one case ($\rho \neq 1$ at tree level) to the other ($\rho = 1$ at tree level). This discontinuity, which has been pointed out previously^{6,5}, is closely tied to the fact that the triplet Higgs boson is non-decoupling^{27,28,10}. Due to this non-decoupling effect, even if the triplet VEV is extremely small, as long as it is non-vanishing, there is the need for the fourth input parameter. The only exception to this is if there is a custodial symmetry which forces $v' = 0$: in this case only the usual three input parameters are necessary.

We also note that the contribution of the lightest neutral Higgs in this case is given by³⁴,

$$(\Delta r)_{LR}^{\text{lightest Higgs}} = \frac{\sqrt{2}G_\mu}{48\pi^2} \left(\frac{M_{W_1}^2}{M_{W_2}^2} \frac{\hat{c}_\theta^2}{\hat{s}_\theta^2} (1 - 2\hat{s}_\theta^2) + \frac{M_{W_1}^2}{M_{Z_2}^2} \frac{1}{\hat{s}_\theta^2} (4\hat{c}_\theta^2 - 1) \right) M_{H^0}^2, \quad (\text{C.8})$$

which depends on M_{H^0} quadratically, and is suppressed by the heavy gauge boson masses, $M_{W_2}^2$ and $M_{Z_2}^2$. The contributions of the remaining scalar fields also have a similar structure.

Appendix D. Minimization of the Scalar Potential in Model with a Triplet

In this section, we summarize our results on minimization of the scalar potential in the model with a triplet Higgs. From the minimization conditions, $\left. \frac{\partial V}{\partial \eta^0} \right|_{\langle H \rangle, \langle \Phi \rangle} =$

$\left. \frac{\partial V}{\partial \phi^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = 0$, we obtain the following conditions,

$$4\mu_2^2 t_\delta + \lambda_2 v^2 t_\delta^3 + 2\lambda_3 v^2 t_\delta - 4\lambda_4 v = 0 \quad (\text{D.1})$$

$$\mu_1^2 + \lambda_1 v^2 + \frac{1}{8}\lambda_3 v^2 t_\delta^2 - \frac{1}{2}\lambda_4 v t_\delta = 0. \quad (\text{D.2})$$

We use the short hand notation, $t_\delta = \tan \delta$. The second derivatives are,

$$\left. \frac{\partial^2 V}{\partial \eta^+ \partial \eta^-} \right|_{\langle H \rangle, \langle \Phi \rangle} = \mu_2^2 + \frac{1}{8}\lambda_2 v^2 t_\delta^2 + \frac{1}{2}\lambda_3 v^2 \quad (\text{D.3})$$

$$\left. \frac{\partial^2 V}{\partial \eta^+ \partial \phi^-} \right|_{\langle H \rangle, \langle \Phi \rangle} = \left. \frac{\partial^2 V}{\partial \eta^- \partial \phi^+} \right|_{\langle H \rangle, \langle \Phi \rangle} = \lambda_4 v \quad (\text{D.4})$$

$$\left. \frac{\partial^2 V}{\partial \phi^+ \partial \phi^-} \right|_{\langle H \rangle, \langle \Phi \rangle} = \mu_1^2 + \lambda_1 v^2 + \frac{1}{8}\lambda_3 v^2 t_\delta^2 + \frac{1}{2}\lambda_4 v t_\delta \quad (\text{D.5})$$

$$\left. \frac{\partial^2 V}{\partial \eta^0 \partial \eta^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = \mu_2^2 + \frac{3}{4}\lambda_2 v^2 t_\delta^2 + \frac{1}{2}\lambda_3 v^2 \quad (\text{D.6})$$

$$\left. \frac{\partial^2 V}{\partial \eta^0 \partial \phi^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = \frac{1}{2}\lambda_3 v^2 t_\delta - \lambda_4 v \quad (\text{D.7})$$

$$\left. \frac{\partial^2 V}{\partial \phi^0 \partial \phi^0} \right|_{\langle H \rangle, \langle \Phi \rangle} = \mu_1^2 + 3\lambda_1 v^2 + \frac{1}{8}\lambda_3 v^2 t_\delta^2 - \frac{1}{2}\lambda_4 v t_\delta. \quad (\text{D.8})$$

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If $\frac{\partial^2 V}{\partial \eta^+ \partial \phi^-} = 0$, then there is no mixing between the doublet and the triplet. This requires $\lambda_4 = 0$.

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